THE STABILITY OF FLOWING TRAINS OF CONFINED RED BLOOD CELLS

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Summary The asymptotic and transient stability of single-file trains of fluid-filled elastic capsules flowing in narrow channels is analyzed as a model for the lines of red blood cells commonly observed in small tubes or vessels. The most amplified disturbances in larger channels are found to have a rich variety of characteristics depending upon the details of the particular configuration. Transient growth mechanisms are found to be significant, even for relatively small perturbations, and are shown to precipitate nonlinear saturation and chaotic flow many times more quickly than the $t \to \infty$ asymptotic stability would predict even for nominally small perturbations.

INTRODUCTION

Red blood cells or similar elastic capsules in sufficiently small vessels or tubes are well-known to flow in a regular single-file formation down the center of the vessel. In wider tubes or vessels, seemingly chaotic flow is observed (*e.g.* figure 1), presumably because such capsule trains are unstable. The source of this instability is unclear yet fundamentally important, particularly how it might be affected by geometric and capsule mechanical properties to avoid line disruption in microfluidic devices to process blood. We consider the character of the most amplifying perturbations that might lead to chaotic flow.

The model system we analyze is a two-dimensional flow of capsules, which empirically displays both stable and chaotic behaviors. We assume that the transition between these regimes arises due to the growth of small perturbations via linear mechanisms. There is no expectation that linearization of this coupled fluid–structure system leads to a diagonalizable system, so we also consider transient linear amplification of disturbances in addition to the eigensystem that governs long-time linear amplification. These methods,[1] as well as the transient non-modal behavior they expose, have been used to study, for example, boundary layer stability. Here they are adapted to the complete fluid–structure coupled flow in the viscous limit. Direct numerical simulations for specific cases confirm both the predicted transient and asymptotic amplification rates, and are used to track the subsequent nonlinear evolution of the system to a chaotic behavior. Of particular interest are the most amplified disturbances and what perturbation amplitudes are needed for transient disturbances to achieve nonlinear saturation significantly before corresponding eigenmodes might lead to finite-amplitude effects at long times, as they must if any eigenvalues are amplifying.

METHODS

The cells are modeled as finite-deformation elastic shells, each containing a Newtonian fluid of area πr_o^2, which for this study matches that of the suspending fluid.[2,3] A boundary integral method[4] is used to evaluate their surface velocities $\mathbf{u}$, which are nonlinear functions of cell surface positions $\mathbf{x}$ due to geometric factors. The full nonlinear system evolves as

$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} = \mathbf{u}(\mathbf{x}). \tag{1}$$

From this numerical model, with M spectral collocation points $\vec{\mathbf{x}}$ representing the cell surfaces, a perturbation method is used to construct the $2M \times 2M$ matrix $\mathbf{A}$ that governs the temporal behavior of perturbations $\vec{\varepsilon}$ to $\vec{\mathbf{x}}$:

$$\frac{\mathrm{d}\vec{\varepsilon}}{\mathrm{d}t} = \mathbf{A}\vec{\varepsilon}. \tag{2}$$

Figure 1: Simulation results for cell shapes and locations for a stable case (left) and an unstable case (right).

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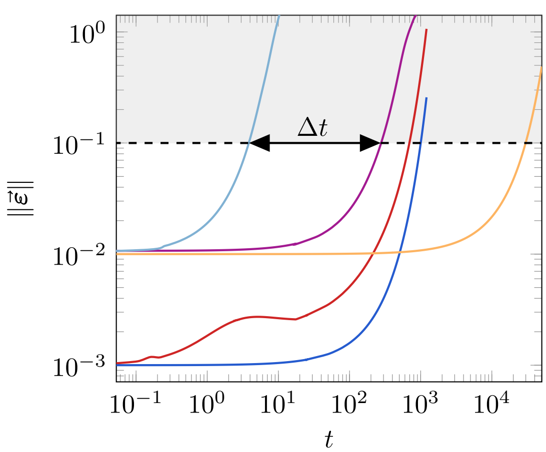
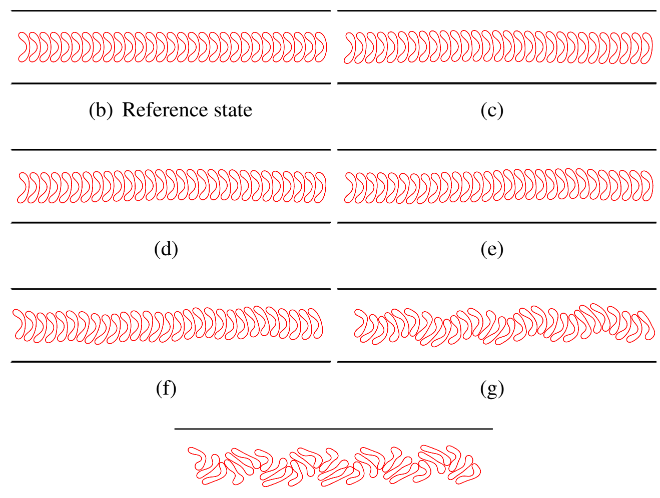
To assess the linear evolution of perturbations governed by (2), we consider both the $t \to \infty$ behavior, dictated by its eigenvalue with the largest positive real component $\alpha = \mathrm{Re}(\lambda_\alpha)$ with corresponding eigenvector $\vec{\mathbf{s}}_\alpha$, and transient growth, which corresponds to the maximum singular value of the singular-value decomposition of $\exp \mathbf{A}t$. The $t \to 0^+$ transient growth rate is η, with corresponding singular vector $\vec{\mathbf{v}}_\eta$. These perturbations, amplification rates, and subsequent transition to nonlinear chaotic motion are considered.

RESULTS

Figure 2 (a) shows the disturbance growth in time for five different initial perturbations for flow of 20 cells in $W = 10r_o$ wide streamwise-periodic channel. The initial disturbances are determined by the linear analysis of $\mathbf{A}$ as outlined and their evolution is computed by direct numerical solution of (1). For initial perturbation amplitudes $\hat{\varepsilon} = 0.001r_o$, the initial transient linear growth rate η is significantly faster than the long-time eigenvalue-based growth rate α. (Direct simulations with small-amplitude initial conditions verify the numerical procedures in the linear limit.) However, for this small initial perturbation, the cumulative amplification of this transiently amplified disturbance is insufficient to lead to significant nonlinear behavior before the eigenvector growth dominates behavior at later times. Increasing the initial perturbation to a still small value of $\hat{\varepsilon} = 0.01r_o$ allows the transient amplification to saturate nonlinearly and develop rapidly to a chaotic flow (see figures 2 b to h). This occurs about 100 times faster than growth at the asymptotically most amplified rate. An ad hoc disturbance for the same amplitude, formed by randomly perturbing cell centroids by the same $\hat{\varepsilon}$ is less amplified still.



(a)



(b) Reference state

(c)

(d)

(e)

(f)

(g)

(h)

Figure 2: (a) Amplification 5 different initial perturbations for a channel with packing $Nr_o/L = 0.7$ and width $W = 40r_o$: — most amplified $t \to 0^+$ disturbance for $\hat{\varepsilon} = 0.001$, — most amplified $t \to \infty$ for $\hat{\varepsilon} = 0.001$; — most amplified $t \to 0^+$ for $\hat{\varepsilon} = 0.01$; — most amplified $t \to \infty$ for with $\hat{\varepsilon} = 0.01$; and — *ad hoc* disturbance $\hat{\varepsilon} = 0.01$. (b) Visualization of the base flow state. (c–h) Evolution of the most unstable eigenvector perturbation to a chaotic flow condition.

The most amplified transient and eigenvector disturbances change qualitatively for different configurations, showing longitudinal displacements, rotations, transverse displacements, and symmetric and asymmetric distortions of the membranes. The corresponding eigenvectors (not shown) are typically different in character from the most amplified transient disturbances.

This work was supported in part by the National Science Foundation under Grant No. CBET 13-36972.

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