BUCKLING AND THE RHEOLOGY OF AN ELASTIC CAPSULE SUSPENSION

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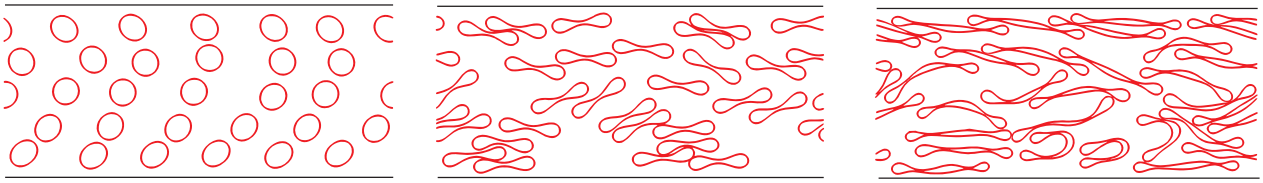
Summary The rheological behavior of an elastic capsule suspension is studied in a model two-dimensional channel using detailed numerical simulations. As the rest capsule membrane aspect ratio increases, the capsules become increasingly vulnerable to a buckling instability. This buckling behavior is concomitant with a sudden increase in the effective viscosity and a near disappearance of any near-wall capsule-free layer. The microstructure dynamics suggest elongated capsules make significant rotational contributions that disrupt organized flow, as computed by their rotlet capsule-capsule interactions.

INTRODUCTION

Flowing elastic capsule suspensions are well-known to have complex properties and rheology. We considered fluid-filled elastic membranes, which can be considered a model for both natural capsules like red blood cells, and artificial ones such as those used for drug delivery. The adjustment of the relative surface area of the capsule membrane leads to expected transitions in rest shape, from round, to biconcave, to elongated, and dog-bone-like. We examine how these shape changes contribute to the complex flow of the confined suspension, whose properties are then quantified by an effective viscosity. In blood or similar complex suspensions, when flowing in narrow confines on the cell scale, the effective viscosity is known to involve the formation of cell-free regions near the vessel wall, thus special attention is afforded to the behavior of this layer for different capsule shapes and its relation to the effective vicosity. This is carried out through the simulation of a two-dimensional simulation model.

METHODS

The flow geometry is a narrow streamwise-periodic two-dimensional channel, for which we vary width W and mean flow rate U. The capsule membranes are finite-deformation linear-elastic shells with finite-deformation bending (M) and tension (T) moduli; each of which contains an incompressible Newtonian fluid with viscosity μ matching that of the suspending fluid. The membrane rest shape is parameterized via $\xi_o = l_o/2\pi r_o$, where l_o is the length of the membrane and πr_o^2 the interior capsule area. We consider several different ξ_o, varying from circular geometries with $\xi_o = 1.0$ to very elongated shapes with $\xi_o = 3.0$. Figure 1 shows the flow of different capsule geometries at capillary number $u^* = \mu U/T = 1.0$.



(a) $\xi_o = 1$ (b) $\xi_o = 1.7$ (c) $\xi_o = 3.0$

Figure 1: Flow visualizations of different ξ_o capsules as indicated flowing in the model channel.

Since the Reynolds number of cell-scale flow is small, inertia is neglected and the governing equations can be solved using a boundary integral formulation.[1] The computation of velocities is accelerated using a particle-mesh-Ewald scheme based on periodic Greens' functions, while channel walls impose a no-slip condition enforced by a penalty method.[2] The membrane positions $\mathbf{x}$ are then advected according to

$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} = \mathbf{u}(\mathbf{x}), \tag{1}$$

where $\mathbf{u}(\mathbf{x})$ are the velocities of the capsule collocation points $\mathbf{x}$ as computed by the boundary integral equation. To evaluate elastic tractions on the fluid, derivatives are calculated using Fourier interpolants of the discrete collocation points representing the capsule membranes. A second-order Runge–Kutta scheme numerically integrates (1).

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RESULTS

We see for the flowing capsules in figure 1, that the (a) circular $\xi_o = 1.0$ and (b) biconcave $\xi_o = 1.7$ cases only slightly deform from their equilibrium configuration, while the (c) $\xi_o = 3.0$ case shows significant folding of some capsules. This seemingly disrupts the otherwise orderly flow, and is considered subsequently as a buckling mechanism.

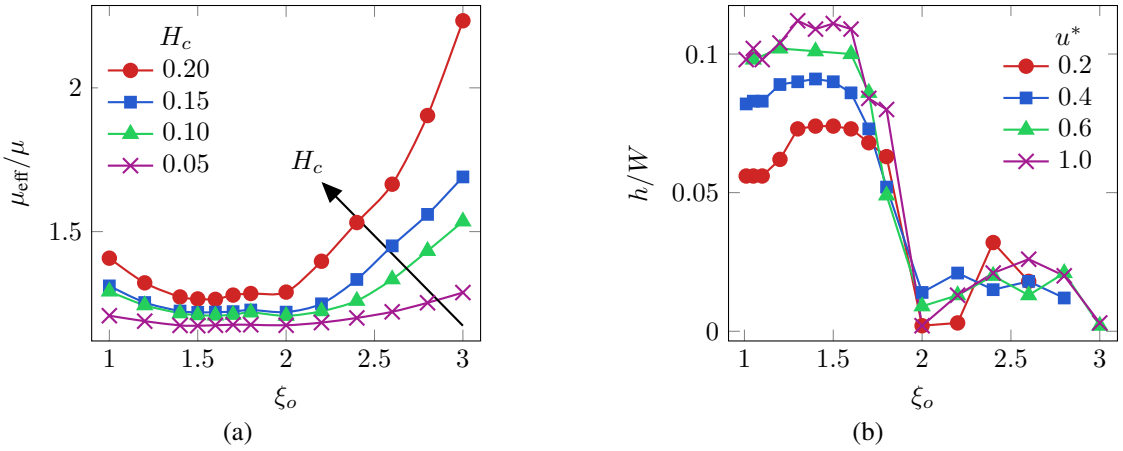

Figure 2: (a) Effective viscosity μ_{eff} for varying equilibrium shapes ξ_o, and (b) corresponding capsule-free layer thickness h.

The effective Newtonian-equivalent viscosities of the suspensions for different area fractions H_c and ξ_o are shown in figure 2 (a). Elongated capsules with $\xi_o \gtrsim 2.0$ have an increasingly large effective viscosity. In blood, the effective viscosity μ_{eff} is reduced through the formation of a cell-free layer. This is measured here as h in figure 2 (b), which decreases to near zero, also for $\xi_o \gtrsim 2.0$, presumably leading to the increase in μ_{eff}.

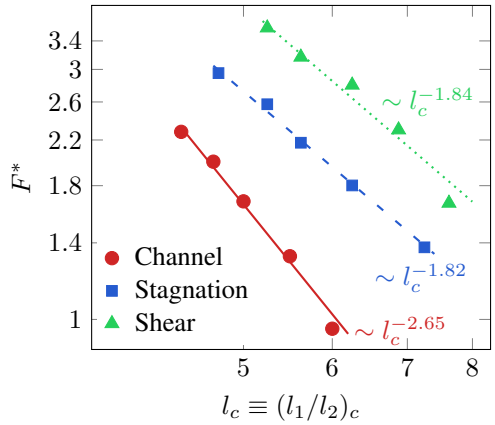

Figure 3: Non-dimensional force F^* versus resting capsule aspect ratio l_c. The straight lines are power-law fits.

We hypothesize that the capsules undergo a buckling transition. Figure 3 shows force $F^* \equiv F/\mu U l_o$ plotted against the aspect ratio $l_c = l_1/l_2$ of the corresponding at-rest capsules. This is compared for single capsules suspended in a Taylor–Green stagnation point flow, in a simple homogeneous shear flow, and in the channel flow. All three show a scaling reminiscent of Euler bucking though altered presumably due to the strong perturbation environment in the channel and the shell-like structure of the capsules.

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References

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